CHAPTER TEN

LOGIC

Statement: This is an assertion or a declaration which may either be true or false. Examples of statements are:

- I. Ghana is in Europe.
- II. Nigeria is a country.
- III. John is a boy.
- IV. A car can travel on the sea.

Conditional statements: These are statements which involve the use of the preposition 'if' and 'then'. Examples of such statements are:

- I. If you steal, then the police will arrest you.
- II. If I am a Nigerian, then I come from Nigeria.
- III. If I am a white man, then I come from Britain.

Conditional statements have the form if P then q, which is abbreviated P=>q. For example, consider the conditional statement: "if you steal, then you will be arrested".

Let P = if you steal

Q = then you will be arrested.

From these we can say that $P \Rightarrow q$ which means that "if you steal, then you will be arrested". Also consider the statement "if I am a white man, then I come from Britain".

Let P = if I am a white man, and q = then I come from Britain. From these, we can say that P = q, which means that "if I am a white man, then I come from Britain".

A conditional statement may either be true or false. For example, the conditional statement, "if I am a human being, then I can fly", is false. But the conditional statement, "if I am a boy, then I am a male" is true.

A conditional statement such as the one just given, which is always true, is referred to as an implication. In the abbreviation $P \Rightarrow q$, P is known as the premise or hypothesis and q is known as the conclusion. For example, consider the conditional statement "if you drink poison, then you will die"

The premise = if you drink poison.

The conclusion = then you will die.

Representation of conditional statements on Venn diagrams:

The conditional statement "if you are a thief, then you are a criminal", is the same as the statement: "All thieves are criminals". Also the statement "if you are a monkey, then you are an animal", is the same as the statement: "All monkeys are animals".

Question 1

Represent each of the following statements on a Venn diagram

a) If you are a blackman, then you come from Africa.

Solution Let B = {Blackmen}. A = {People from Africa}.

b) "if you are a thief, then you are a criminal".

Solution Let $C = \{Criminals\}.$ $T = \{Thieves\}.$



c) "All my friends are good boys".

N/B: The statement implies that if you are my friend, then you are a good boy.

Solution Let F = {My friends}. C = {Good boys}.



Question 2 Analysis 1

If the statement says that Esi sits on the front row, then Esi can only be found anywhere within the shaded portion of the next diagram.



Let $U = \{$ Students in the class $\}$.

 $G = {girls}.$

 $F = \{$ those who sit on the front row $\}.$

Since Esi can be found anywhere within the shaded portion, then the following conclusions can be drawn:

- a) Esi sits on the front row, since she must be found within the shaded portion.
- b) Since the shaded portion where Esi can be found lies within the set G, which represents the set of girls, then we can also conclude that Esi is a girl.
- c) Finally, since the shaded portion also lies within the universal set U, which represents the set of students within the class, then our final valid or true conclusion which can be made is that Esi is a student within the class.

Analysis 2

Assuming that the statement says that Esi is a girl, then she can be found anywhere within the shaded portion.



Since the shaded portion where Esi can be found lies within the set U, then we can draw the valid conclusion that Esi is a student in the class. Since Esi can be found at any portion within the shaded portion, then Esi may also be found in the set F or the triangle.

Therefore for the fact that Esi is a girl does not mean that she must be found within the set F. N/B: Even though in the given answers to the questions, the circle was the symbol used, other geometrical symbols such as triangles, rectangles and squares could have been used.

Logical Analysis:

In solving questions based on logic, the question must be well analyzed from the logical point of view, and not from the normal point of view. Even though a conclusion may seem right from the normal point of view, it may not be correct from the logical point of view.

Analyzing a statement from the logical point of view, may give us a different meaning from the normal point of view. Certain questions on logic can only be solved, by subjecting them to logical analysis. The logical analysis we are going to make next will be centred on the statement: "All those who sit on the front row are girls, and Esi is a girl".

Example 1

Illustrate the following information on a Venn diagram:

- 1. All those who sit on the front row are girls.
- 2. Esi is a girl.

Solution

Let $U = \{$ Students in the class $\}$.

- $F = \{$ Those who sit on the front row $\}.$
- $G = {Girls}.$
- **.** = Esi.



N/B: This Venn diagram is that for the statement: "All those who sit on the front row are girls and Esi is a girl. All that it means is that Esi can be found within the set F or outside the set F, i.e. she must be found within the set G and for this reason to draw the conclusion that because Esi is a girl, then she must sit on the front row i.e. found within the set F will be wrong, invalid or false. The right conclusion to draw is that Esi may sit on the front row or not.

N/B: If Esi is a girl, then she can be found at the position inside the triangle indicated by the point P, and as such sit on the front row. Also if Esi is a girl, then she can also be found at positions outside the triangle indicated by the points x and y, due to which she cannot sit on the front row.

Analysis 3

Assuming the statement says that Esi is not a girl, then Esi can be found anywhere within the shaded portion of the next diagram, i.e. outside the set G.



If Esi can only be found within the shaded portion, then she can never be found within the set F or the triangle. Therefore the statement "if Esi is not a girl, then she can never sit on the front row", will be valid or true.

Since the shaded portion lies within the universal set U, then another valid conclusion we can draw is that, Esi is a student within the class.

Analysis 4

If the statement says that Esi does not sit on the front row, then Esi can be found anywhere within the shaded portion, indicated in the next diagram. In short, she must be found only outside the set F.



Esi can be found within the set G, at the position indicated by the point w, in which case, she will be a girl, or can also be found at the position indicated by the point R outside the set G or the circle, for which case she is not a girl.

Therefore to conclude that for the fact that Esi does not sit on the front, then she must be a girl or Esi is a girl, will be invalid.

The right or true or valid conclusion to draw is that Esi can be a girl or not a girl, or in short, Esi may possibly be a girl. Also to draw the conclusion that because Esi does not sit on the front row, then she cannot be a girl will also be invalid, since she can be found at the point represented by the point w, for which she can be a girl.